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An Algorithm for Group Decision Making Problems via Soft Set Relations

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Research Article

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Abstract
Decision-making is a critical process in various fields, often complicated by uncertainty and incomplete information. Soft set theory, introduced as a mathematical tool to handle such uncertainties, offers a flexible and efficient framework for addressing these challenges. The purpose of this study is developing a method by using graph representation of soft set relation for group decision making problems with several examples as application. The algorithm integrates key soft set relations, allowing for the systematic evaluation of alternatives against multiple criteria. Through a some of examples and case studies, effectiveness of the algorithm in decision-making applications is examined. Results show that the soft set-based approach provides a more adaptable and precise method for decision-making compared to traditional techniques, making it a valuable tool for complex decision environments.

Keywords: Soft sets, soft set relations, group decision making problems, graph representations of soft set relations.

Grup Karar Verme Problemleri için Esnek Küme Bağıntılarıyla Bir Algoritma

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Öz
Karar verme, çeşitli çalışma alanlarında genellikle belirsizlik ve eksik bilgi nedeniyle karmaşık hale gelen kritik bir süreçtir. Bu tür belirsizliklerle başa çıkmak için matematiksel bir araç olarak tanıtılan esnek küme teorisi, bu zorlukları ele almak için esnek ve etkili bir çerçeve sunar. Bu çalışmanın amacı, uygulama olarak birkaç örnekle grup karar verme problemleri için esnek küme ilişkisinin graf gösterimini kullanarak bir yöntem geliştirmektir. Algoritma, alternatiflerin birden fazla kritere göre sistematik olarak değerlendirilmesine olanak tanıyan temel esnek küme bağıntılarını kullanır. Örnekler aracılığıyla, algoritmanın karar verme uygulamalarındaki sağlamlığını ve etkinliğini gösterilmiştir. Sonuçlar, esnek küme tabanlı yaklaşımın, geleneksel tekniklere kıyasla karar verme için daha uyarlanabilir ve kapsamlı bir yöntem sağladığını ve karmaşık karar verme problemleri için değerli bir araç haline getirdiğini göstermektedir.

Anahtar Kelimeler: Esnek kümeler, esnek küme bağıntıları, grup karar verme problemleri, esnek küme bağıntılarının grafik gösterimi.

Introduction

Soft sets are a powerful tool in group decision-making, especially when dealing with uncertainty and incomplete information. Unlike classical mathematical tools, it provides a more flexible and adaptable approach to dealing with ambiguous and incomplete information. Soft set theory, that is introduced by Molodtsov in 1999 [1], is used to model uncertainty without the need for precise data. In group decision-making, soft sets have been extended to various hybrid models that incorporate fuzzy logic, hesitant fuzzy sets, and other complex structures to improve decision-making processes. Some of the studies using the advantages of soft sets can be listed as follows: First study was developed by Maji et al. [2] who employed the binary table with the rows and columns that are labeled by objects and parameters, respectively. Here, a value of 1 signifies that an object belongs to $F(e)$, while 0 indicates that it does not. This tabular representation is used to depict soft sets (SSs). For the selection phase, the authors developed the Simple Additive Weighting (SAW) method for soft sets and introduced the concept of a choice value for each object. Their proposed algorithm enables the ranking of objects based on the total number of parameters they satisfy. Chen et al. designed an algorithm for parameter reduction in the paper [3], Çağman and Enginoglu in [4] used an operation on soft sets called the And Product. To rank the objects, they introduced a novel method known as the uni-int approach. This method involves the union of intersections derived from the soft set obtained through the AND-Product of the soft sets (SSs). The uni-int decision function is then utilized to determine the final decision set. Çağman and Enginoğlu [5] introduced the concept of a soft matrix, an mxn matrix whose entries consist of 0s and 1s where m refers to the carnality of the universal set of objects and n refers to the number of parameters. Soft graphs are applied to the decision making problems by Hayat et al. [6] For more papers for the literature review of soft set based decision making models the papers written by Danjuma et al. [7], Khameneh et al. [8] and Alcantud et al. [9] would be helpful for readers. In papers [7-9], not only systematic reviews on decision-making based on soft set were stated but also hybrid models with soft set theory are listed with advantages and disadvantages. More studies and researches can be listed as [10-15], about soft sets. Soft set theory, with its ability to manage uncertainty and imprecision, has seen substantial development in recent years. Another important topic is representaions of soft sets. Among these advancements, reduced soft matrices and generalized products, as introduced by Atagün et al. [16] provide a streamlined approach for solving complex decision-making problems. This methodology simplifies the representation of soft sets while maintaining their structural integrity, enhancing computational efficiency in practical applications. Building on this foundation, Kamacı et al. [17] explored row-products of soft matrices, demonstrating their effectiveness in multiple-disjoint decision-making scenarios. This technique enables the simultaneous evaluation of distinct decision criteria, making it a powerful tool in fields requiring robust and adaptable decision-making frameworks. Further advancements have been made through the cardinality inverse soft matrix theory, which Kamacı and collaborators [18] employed to address multicriteria group decision-making challenges. This approach

emphasizes the interplay between soft set cardinalities and matrix operations, offering novel solutions for problems where multiple decision-makers and criteria are involved. Atagün and Kamacı [19] extended these concepts by introducing decompositions of soft sets and matrices, further broadening the applicability of soft set theory in group decision-making contexts. These decompositions enable a more granular analysis of decision-making scenarios, facilitating the identification of optimal solutions in complex environments. In parallel, Hayat et al. [6, 20] focused on type-2 soft sets and their graph-based representations, uncovering new perspectives on uncertainty modeling. The introduction of type-2 soft graphs provides a versatile framework for characterizing relationships within uncertain datasets, with applications spanning network analysis and social decision-making. The characterization of these graphs [20] enriches the theoretical underpinnings of soft set theory, offering insights into their structural properties and enhancing their utility in real-world applications. Collectively, these studies underscore the versatility and evolving potential of soft set theory and its matrix-based extensions in addressing modern decision-making challenges. Many decision-making methods ignore the priority order of the decision maker's preferences. The advantage of the method presented in this study over others that are listed above is that it allows for the creation of a priority order for the decision-makers' preferences. Moreover one of the main tools in this study is graph which are used to represent soft set relations. In previous studies a similar approached has been done by Polat et al. [15] but only for single decision maker. Unlike the method given by Polat et al. [15], recommended method in this study is designed for multi observer decision making methods. The method which was given in paper [15] is improved and to the group decision making problems by using soft set relation properties. In this section basic definitions on soft set theory is reminded.

Definition 1. [1] Assume that U be an initial universal set and E be a set of parameters, $P(U)$ denote the power set of U. A pair (S, E) is called a soft set over U , where is S is a mapping given by $S : E \rightarrow P(U)$. Henceforth the abbreviation SS will be used to symbolize the soft sets.

Definition 2. [10] Let (S_1, E_1) and (S_2, E_2) be SSs over a mutual universe U,

i) The definition of the product set of two SSs (S_1, E_1) and (S_2, E_2) is given as $(S_1, E_1)x(S_2, E_2) = (\mathrm{H}, E_1 \times E_2)$ where $H : E_1 \times E_2 \rightarrow P(U \times \mathrm{U})$ where $H(x, y) = F(x) \times G(y)$ for all $(x, y) \in E_1 \times E_2$.

ii) A soft set relation R from (S_1, E_1) to (S_2, E_2) is a SS of $(S_1, E_1)x(S_2, E_2)$.

Definition 3. [10] Assume that Q and R be soft set relations on the SS (S, E). The set $Q \cap R = \{\mathrm{S(a)} \times \mathrm{S(b)} \in \mathrm{Q} \text{ and } \mathrm{S(a)} \times \mathrm{S(b)} \in \mathrm{R}\}$ which is denoted by $Q \cap R$ represents the intersection of two soft set relation Q and R.

General Methods

Representing A Soft Set Relation by Directed Graphs and Matrices

This section provides fundamental definitions in graph theory, as outlined in the book which was written by Grimaldi in 2001 [21].

Definition 4. [21] A directed graph $G = (V, E)$ is composed of a set V of vertices (or nodes) and a set E of edges. Some basic definitions related to graphs are as follows:

- A vertex $a \in V$ is referred to as the initial vertex of the edge $(a, b) \in E$, while vertex $b \in V$ is known as the terminal vertex of the same edge.
- An edge of the form $(a, a) \in E$ is called a loop.
- A path exists from $a \in V$ (where $b \in V$, $(a \neq b)$, and this path consists of a finite sequence of directed edges.

Definition 5. [21] If $G = (V, E)$ is a graph, then $G_1 = (V_1, E_1)$ is called a subgraph of G if $\emptyset \neq V_1 \subseteq V$ and $\emptyset \neq E_1 \subseteq E$ where each edge in G_1 is incident with vertices in V_1.

A soft set relation can be represented using a directed graph. The number of vertices in the graph is equal to the number of elements in the set from which the soft set relation has been defined. For each ordered pair $(S(a), S(b))$ in the relation R, there will be a directed edge from the vertex '$S(a)$' to vertex '$S(b)$'. If there is an ordered pair $(S(a), S(a))$, there will be a self- loop on vertex '$S(a)$'.

Soft set relations defined on a finite parameter set can be represented using specialized matrices, much like the classical relations discussed in Grimaldi's book [16].

Definition 6. [11] Consider the labeled parameter sets $A_1 = \{a_1, a_2, \ldots, a_n\}$ and $A_2 = \{b_1, b_2, \ldots, b_m\}$ with n and m elements respectively. Any soft set relation $Q = \{S_1(a_i) \times S_2(b_j) : a_i \in A_1, b_j \in A_2\}$ from and (S_1, A_1) to (S_2, A_2) can be represented by a Matrix called $M_Q = [Q_{ij}]$ where

$$r_{ij} = \begin{cases} q_{ij} = 1 & \text{if} \quad S_1(a_i) \text{ and } S_2(b_j) \text{ are related,} \\ q_{ij} = 0 & \text{if} \quad S_1(a_i) \text{ and } S_2(b_j) \text{ are not related} \end{cases}$$

Theorem 1. [11] Assume that Q and R be a soft set relations whose matrix representations are given as $M_Q = [q_{ij}]$ and $M_R = [r_{ij}]$ from the soft sets (S_1, A_1) to (S_2, A_2), where A_1, A_2 be the parameter sets with m, n elements respectively.

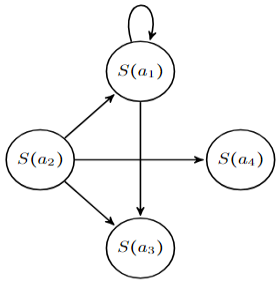
The matrix representation of soft set relation $Q \cap R$ is $M_Q \cap M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} m_{ij} = 1 \text{ if } q_{ij} = 1 \text{ and } r_{ij} = 1, \\ m_{ij} = 0 \text{ if } q_{ij} = 0 \text{ or } r_{ij} = 0. \end{cases}$$

The theorem given above will be quite useful when finding the intersections of relations. It can also be used in subsequent sections to simplify the steps of the algorithm.

Example 1. Let (S, E) be a SS over U where $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $E = \{a_1, a_2, a_3, a_4\}$ with set valued function $S(a_1) = \{x_1\}$, $S(a_2) = \{x_1, x_4, x_5, x_6\}$, $S(a_3) = \{x_2, x_4, x_5, x_6\}$ and $S(a_4) = \{x_5, x_6\}$. Define the soft set relation on (S, E) as $R = \{S(a_1) \times S(a_1), S(a_2) \times S(a_1), S(a_2) \times S(a_3), S(a_1) \times S(a_3), S(a_2) \times S(a_4)\}$.

This soft set relation R can be pictured by directed graph G_R and matrix M_R:



$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1. *Graph and Matrix representation of soft set relation S.*

In this representations the set of vertices $V = \{S(a_1), S(a_2), S(a_3), S(a_4)\}$, and the set of edges

$$E = \{(S(a_1), S(a_1)), (S(a_2), S(a_1)), (S(a_2), S(a_3)), (S(a_1), S(a_3)), (S(a_2), S(a_4))\}.$$

The graph representations of soft set relations were previously used in a decision-making problem modeling in the article titled ”A method for decision making problems by using graph representation of soft set relations” that was written by [15] in 2019. In this study, the algorithm presented in [15], will be further developed and adapted to group decision-making methods via these two representation of Soft set relations.

Results and Discussion

Revised Algorithm for Group Decision Making Problems

Group decision-making (GDM) involves multiple individuals working together to make decisions, which can be complex due to differing opinions, preferences, and objectives. To handle uncertainties and vagueness inherent in human judgments, soft sets, introduced by Molodtsov in 1999, have gained prominence. Soft sets provide a flexible mathematical tool to deal with uncertainty without requiring complex parameterization. The method described in Article 15 was designed for decision-making problems involving a single decision-maker. However, in this study, that method has been extended to address decision-making problems involving multiple decision-makers simultaneously. In the developed approach, the individual preference rankings of each decision-maker have been taken into consideration. Based on the individual rankings of each user, relationships derived from the ranking of parameters have served as the foundation of the algorithm proposed for this decision-making problem. Expanding on this, the proposed method aims to enhance the applicability of the previously designed single-decision-maker algorithm to a multi-decision-maker context. This involves accounting for the

diverse perspectives and rankings of multiple decision-makers, ensuring that their individual preferences are systematically integrated into the decision-making process. The relationships derived from these rankings form the backbone of the methodology, enabling the algorithm to handle complex decision-making scenarios that require harmonizing multiple viewpoints.

1. Construct the Soft set relation from given priorities for each judges.
2. Find the pairwise intersections of relations which were obtained from the first step.
3. Construct the graph representations or matrix representations of derived soft set relations.
4. Determine the number of incoming and outgoing paths for each vertex (In Matrix representation the value of incoming ways is the row sum of the corresponding vertex and the value of outgoing ways is the column sum of the corresponding vertex) and write them as ordered pairs for each relation, where the first component indicates the number of incoming paths and the second component indicates the number of outgoing paths.
5. Analyze the images of each set to identify the presence of elements from the universal set. To do this, treat the images as vectors, where each element represents the presence of corresponding elements from the universal set in the same order.
6. Evaluate the total incoming and outgoing values for each $S(a_1)$ by adding the numbers of incoming and outgoing paths to each component of the vectors. If $S(a_1)$ is in the intersection of relation weight of this component will be 2. If $S(a_1)$ is in more than one intersection of relation then the weight of this component would be the number of the intersection of relations.
7. Calculate the final values of incoming and outgoing for each object. The object with the highest incoming value represents the optimal choice. If multiple objects share the highest incoming value, the object among them with the highest outgoing value will be the final choice.

To clarify the algorithm following example is given and every steps of algorithm is explained in detail.

Example 2. A two-partner company needs to relocate its headquarters to a new office. The management partner Mr. X and Mr. Y must choose between four potential locations:

Location A: Downtown City Center,

Location B: Suburban Business Park,

Location C: Emerging Tech Hub Area,

Location D: Established Industrial District.

Each location has advantages and disadvantages. The list of parameters that Mr. X and Mr. Y are looking for: Cost, Accessibility, Employee Preferences, Safety and Security and Proximity to Clients. According to the Scenario the sets of universe and parameters are U = {Location $A = \alpha_1$, Location $B = \alpha_2$, Location $C = \alpha_3$, Location $D = \alpha_4$},

E={Accessibility=e_1, Employee Preferences=e_2, Proximity to Clients=e_3, Safety and Security=e_4, Expensive=e_5}.

The soft set on the universal set U is gives as

$S(e_1) = \{\alpha_1, \alpha_2, \alpha_4\}$.

$S(e_2) = \{\alpha_2, \alpha_4\}$.

$S(e_3) = \{\alpha_1\}$.

$S(e_4) = U$.

$S(e_5) = \{\alpha_1, \alpha_2, \alpha_3\}$.

The priority order of Mr. X: Expensive, Accessibility, Employee Preferences, Proximity to Clients, Safety and Security respectively.

The priority order of Mr. Y: Safety and Security, Accessibility, Expensive, Proximity to Clients, Employee Preferences respectively.

Application of Method:

Step 1: The soft set relations R_1 and R_2 according to given priorities for Mr.X and Mr. Y are respectively

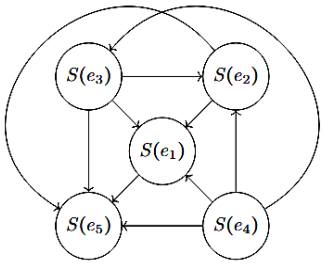
$$R_1 = \{S(e_1) \times S(e_5), S(e_2) \times S(e_5), S(e_3) \times S(e_5), S(e_4) \times S(e_5), S(e_2) \times S(e_1), S(e_3) \times S(e_1), S(e_4) \times S(e_1), S(e_3) \times S(e_2), S(e_4) \times S(e_2), S(e_4) \times S(e_3)\}.$$

$$R_2 = \{S(e_1) \times S(e_4), S(e_5) \times S(e_4), S(e_3) \times S(e_4), S(e_2) \times S(e_4), S(e_5) \times S(e_1), S(e_3) \times S(e_1), S(e_2) \times S(e_1), S(e_3) \times S(e_5), S(e_2) \times S(e_5), S(e_2) \times S(e_3)\}.$$

Step 2: Intersection of soft set relations R_1 and R_2 is

$$R_1 \cap R_2 = \{S(e_3) \times S(e_5), S(e_2) \times S(e_5), S(e_2) \times S(e_1), S(e_3) \times S(e_1)\}.$$

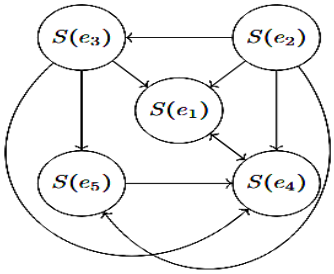
Step 3: The graph representations of relations R_1 and R_2 are given below.



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2. *Graph and Matrix representation of the soft set relation* R_1

Similarly graph representation and matrix representation of soft set relation R_2 are gives as follows



$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 3. *Graph and Matrix representation of the soft set relation* R_2

Step 4: In the graph 1 for the node $S(e_1)$ there is 3 incoming and 1 outgoing ways. Thus we get pair (3,1) for the node $S(e_1)$. The list of all ways given in below:

$S(e_1)$ (3,1)
$S(e_2)$ (2,2)
$S(e_3)$ (1,3)
$S(e_4)$ (0,4)
$S(e_5)$ (4,0)

The list of all ways derived from the Graph 2 given in below:

$S(e_1)$ (3,1)
$S(e_2)$ (0,4)
$S(e_3)$ (1,3)
$S(e_4)$ (4,0)
$S(e_5)$ (2,2)

Step 5:

$(1,1,0,1)$ for the node $S(e_1)$
$(0,1,0,1)$ for the node $S(e_2)$
$(1,0,0,0)$ for the node $S(e_3)$
$(1,1,1,1)$ for the node $S(e_4)$
$(1,1,1,0)$ for the node $S(e_5)$

Step 6:

For the first graph

$(6,6,0,6)$ and $(2,2,0,2)$ for the node $S(e_1)$
$(0,4,0,4)$ and $(0,4,0,4)$ for the node $S(e_2)$
$(2,0,0,0)$ and $(6,0,0,0)$ for the node $S(e_3)$
$(1,1,1,1)$ and $(5,5,5,5)$ for the node $S(e_4)$
$(7,7,7,0)$ and $(1,1,1,0)$ for the node $S(e_5)$

For the second graph

$(6,6,0,6)$ and $(2,2,0,2)$ for the node $S(e_1)$
$(0,2,0,2)$ and $(0,6,0,6)$ for the node $S(e_2)$
$(2,0,0,0)$ and $(6,0,0,0)$ for the node $S(e_3)$
$(5,5,5,5)$ and $(1,1,1,1)$ for the node $S(e_4)$
$(5,5,5,0)$ and $(3,3,3,0)$ for the node $S(e_5)$

Step 7: For Mr. X Total incoming and outgoing values for each objects respectively $\alpha_1 = (16,14)$, $\alpha_2 = (18,12)$, $\alpha_3 = (8,6)$,and $\alpha_4 = (11,11)$,and For Mr. Y Total incoming and outgoing values for each objects respectively $\alpha_1 = (18,12)$, $\alpha_2 = (18,12)$, $\alpha_3 = (10,4)$,and $\alpha_4 = (13,9)$. The choice object is α_2.

Example 3. A company is looking to replace its outdated project management software with a new one. The decision involves various departments like IT, finance, marketing, and human resources, each with different priorities. The sets of different project management softwares is universal set that is denoted by $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. The set of parameters $E = \{a_1$=security, a_2= ease of integration with

existing systems, a_3=costly, a_4= collaboration, a_5=enhance creativity, a_6=user-friendly , a_7=affordable$\}$. The soft set on the universal set U is given as:

$S(a_1) = \{x_1, x_2, x_3, x_4, x_6, x_7\}$.

$S(a_2) = \{x_2, x_3, x_4, x_6\}$.

$S(a_3) = \{x_1\}$.

$S(a_4) = \{x_1, x_2, x_3\}$.

$S(a_5) = \{x_2, x_3, x_4, x_6\}$.

$S(a_6) = \{x_1, x_5\}$.

$S(a_7) = \{\}$.

The prioritizes of each department are listed below respectively.

● IT Department prioritizes security, ease of integration with existing systems, user-friendly, enhance creativity, collaboration. i.e. $a_1 > a_2 > a_7 > a_6 > a_5$.

● Finance prioritizes cost, user-friendly, security, collaboration, ease of in-

tegration with existing systems. i.e. $a_3 > a_6 > a_1 > a_4 > a_2$.

● Marketing prioritizes enhance creativity, user-friendly, security, collaboration, ease of integration with existing systems, cost. i.e. $a_5 > a_6 > a_1 > a_4 > a_2 > a_3$.

● HR needs a user-friendly enhance creativity, collaboration, security, ease of integration with existing systems, cost. i.e. $a_6 > a_5 > a_4 > a_1 > a_2 > a_3$.

The corresponding soft set relations for the departments IT, Finance, Marketing and HR are labelled as R_1, R_2, R_3 and R_4 respectively.

$R_1 = \{S(a_2)\times S(a_1), S(a_7)\times S(a_1), S(a_6)\times S(a_1), S(a_5)\times S(a_1), S(a_7)\times S(a_2), S(a_6)\times S(a_2), S(a_5)\times S(a_2), S(a_6)\times S(a_7), S(a_5)\times S(a_7), S(a_5)\times S(a_6)\}$.

$R_2 = \{S(a_6)\times S(a_3), S(a_1)\times S(a_3), S(a_4)\times S(a_3), S(a_2)\times S(a_3), S(a_1)\times S(a_6), S(a_4)\times S(a_6), S(a_2)\times S(a_6), S(a_4)\times S(a_1), S(a_2)\times S(a_1), S(a_2)\times S(a_4)\}$.

$R_3 = \{S(a_6)\times S(a_5), S(a_1)\times S(a_5), S(a_4)\times S(a_5), S(a_2)\times S(a_5), S(a_3)\times S(a_5), S(a_1)\times S(a_6), S(a_4)\times S(a_6), S(a_2)\times S(a_6), S(a_3)\times S(a_6), S(a_4)\times S(a_1), S(a_2)\times S(a_1), S(a_3)\times S(a_1), S(a_2)\times S(a_4), S(a_3)\times S(a_4), S(a_3)\times S(a_2)\}$.

$R_3 = \{S(a_6)\times S(a_5), S(a_1)\times S(a_5), S(a_4)\times S(a_5), S(a_2)\times S(a_5), S(a_3)\times S(a_5), S(a_1)\times S(a_6), S(a_4)\times S(a_6), S(a_2)\times S(a_6), S(a_3)\times S(a_6), S(a_4)\times S(a_1), S(a_2)\times S(a_1), S(a_3)\times S(a_1), S(a_2)\times S(a_4), S(a_3)\times S(a_4), S(a_3)\times S(a_2)\}$.

$R_4 = \{S(a_5)\times S(a_6), S(a_4)\times S(a_6), S(a_1)\times S(a_6), S(a_2)\times S(a_6), S(a_3)\times S(a_6), S(a_4)\times S(a_5), S(a_1)\times S(a_5), S(a_2)\times S(a_5), S(a_3)\times S(a_5), S(a_1)\times S(a_4), S(a_2)\times S(a_4), S(a_3)\times S(a_4), S(a_2)\times S(a_1), S(a_3)\times S(a_1), S(a_3)\times S(a_2)\}$.

Graph representations G_1, G_2, G_3 and G_4 for each relations R_1, R_2, R_3 and R_4 are listed in following chart respectively.

There is 6 new soft set relation derived from Pairwise intersection of relations R_1, R_2, R_3 and R_4. The matrix representations of the relations will be useful in finding their pairwise intersections. The table below shows the pairwise intersections of the matrices.

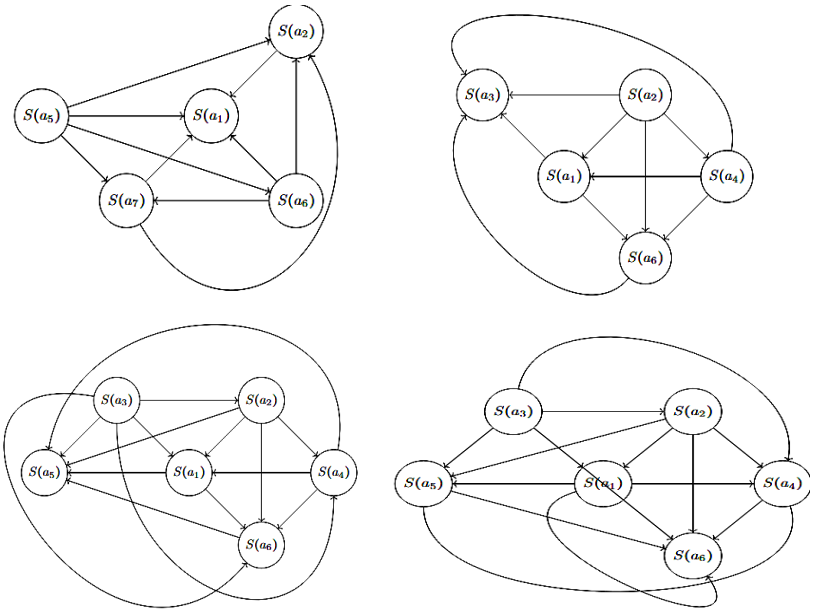


Figure 4. *Graph representations of relations R_1, R_2, R_3 and R_4.*

$$M_{R_1} \cap M_{R_2} = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 1&1&0&0&0&1&1 \\ 1&1&0&0&0&0&1 \\ 1&1&0&0&0&0&0 \end{bmatrix} \quad M_{R_1} \cap M_{R_3} = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_{R_1} \cap M_{R_4} = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} \quad M_{R_2} \cap M_{R_3} = \begin{bmatrix} 0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_{R_2} \cap M_{R_4} = \begin{bmatrix} 0&0&0&0&0&1&0 \\ 1&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} \quad M_{R_3} \cap M_{R_4} = \begin{bmatrix} 0&0&0&0&1&1&0 \\ 1&0&0&1&1&1&0 \\ 1&1&0&1&1&1&0 \\ 0&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix}$$

Figure 5. *Matrix representations of relations R_1, R_2, R_3 and R_4.*

For R_1, $(5,5,5,5,0,5,5)$ and $(1,1,1,1,0,1,1)$,

For R_2, $(0,4,4,4,0,4,0)$ and $(0,2,2,2,0,2,0)$,

For R_3, $(0,4,4,4,0,4,0)$ and $(0,2,2,2,0,2,0)$,

Final incoming and outgoing values for each objects according to the relations R_1, R_2, R_3 and R_4 and their impact of pairwise intersections to each element are given in the below respectively.

For x_1 (9,7) (14,10) (13,15) (14,14),

For x_2 (11,9) (7,13) (15,13) (14,14),

For x_3 (11,9) (7,13) (15,13) (14,14),

For x_4 (10,8) (5,9) (12,9) (10,11),

For x_5 (2,4) (4,2) (5,0) (6,1),

For x_6 (10,8) (5,9) (12,9) (10,11),

And For x_7 (5,1) (3,3) (4,3) (3,4).

Conclusion

Finding solutions to decision-making problems continues to be a highly engaging research area, attracting the attention of many researchers. Numerous studies in this field utilize a variety of approaches and theories. One particularly versatile and practical theory is Soft Set theory, which has often been combined with other existing frameworks, such as Fuzzy Sets, Interval Sets, and Rough Sets, to enhance its applicability. Unlike previous studies, where Soft Sets served as a standalone tool for solving decision-making problems, this study emphasizes their integration with the priority rankings of decision-makers and offers a more robust and comprehensive approach to complex decision-making scenario. Future studies could further explore the potential of Soft Set theory in decision-making by integrating additional theories or frameworks to address even more complex, multi-dimensional problems. Additionally, investigating the dynamic interaction of decision-makers' preferences over time and the development of adaptive algorithms could open new avenues for research in collaborative decision-making environments.

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